

$$6.1 (i) F(E) = [1 + \exp(E - E_F)/kT]^{-1} \quad (1)$$

For $E - E_F = kT$, $F(E) = (1 + e)^{-1} = 0.270$

(ii) When $E - E_F = 0.5 \text{ eV}$ and $F(E) = 1\%$,

$$\text{from eqn (1), } T = \frac{E - E_F}{k \log(1/F - 1)} = \frac{0.5}{8.62 \times 10^{-5} \log(1/0.01 - 1)} = 1262.3 \text{ K}$$

6.3 The energy of the incident light = hc/λ

$$= 6.62 \times 10^{-34} \times 3 \times 10^8 / 0.2 \times 10^{-6} \text{ J} = 6.2 \text{ eV}$$

Since this energy is larger than the work functions of all the metals listed in [Table 6.2], all of them will emit electrons in response to the input light.

6.4 The density of occupied states = density of states $\times$ Fermi function

At an energy kT above Fermi level E_F ,

the density of occupied states: $C\sqrt{E_F + KT} / (1 + e)$

where C is a constant given by [eqn 6.10].

For the same density of occupied states at an energy E below the Fermi level, $E' = E_F - E$

$$\sqrt{E_F - E} / [1 + \exp(-E/kT)] = \sqrt{E_F + kT} / (1 + e) \quad (1)$$

Since the change of the density of states is much slower than the change in Fermi function for small E , the solution of eqn (1) must be at a large value of E , so that the Fermi function becomes fairly constant and the density of states can compensate the difference due to the change in the Fermi function. Hence eqn (1) can be written approximately as

$$E \gg kT, \exp(-E/kT) \rightarrow 0$$

$$\sqrt{E_F - E} \times 1 = \sqrt{E_F + kT} / (1 + e)$$

$$E_F \gg kT$$

Giving $E = 0.928 E_F$; thus $E' = 0.072 E_F$

6.8 The number of photons in the laser beam = $\frac{\text{laser power}}{\text{energy of a photon}} \text{ s}^{-1}$

$$= 2 \times 10^{-3} / \left(\frac{hc}{\lambda} \right) = \frac{2 \times 10^{-3} \times 632.8 \times 10^{-9}}{6.62 \times 10^{-34} \times 3 \times 10^8} = 6.37 \times 10^{15} \text{ s}^{-1}$$

Each photon induces 10^{-4} electrons. Thus the total current induced by the laser beam = $6.37 \times 10^{15} \times 10^{-4} \times 1.6 \times 10^{-19}$ coulomb per second = 1.02×10^{-2} A. Assume that all the electrons come from the Fermi level. Then they come out with a kinetic energy $E_{KE} = hf - \phi$. The electron flow may thus be stopped with an anode voltage so negative that $-eV_a = E_{KE}$. Hence, the work function may be obtained from $f = hf - |eV_a|$.

6.9 no. of Cu atoms in 1 m^3 , $N_a = \frac{9.4 \times 10^3}{63.5} \times 6.02 \times 10^{26} = 8.91 \times 10^{28}$

Each Cu atom contributes one electron, therefore $N_e = 8.91 \times 10^{28} \text{ m}^{-3}$.

Hence $E_F = \frac{h^2}{2m} \left(\frac{3N_e}{8\pi} \right)^{2/3} = 7.27 \text{ eV}$

Electronic specific heat = $\frac{\pi^2 k^2 T}{2E_F} N_e$ / density of Cu (from [eqn 6.25])
 $= 2.24 \text{ J kg}^{-1} \text{ K}^{-1}$ (taking room temp. as 293 K)

Lattice specific heat = $3N_e k$ / density of Cu
 $= 3.92 \times 10^2 \text{ J kg}^{-1} \text{ K}^{-1}$

So the total specific heat = $2.24 + 3.92 \times 10^2 = 394 \text{ J kg}^{-1} \text{ K}^{-1}$, and $2.24/394$; 0.5% is contributed by the electrons.

In reference books, they give a value of about $398 \text{ J kg}^{-1} \text{ K}^{-1}$ which is close to our estimation.